Turbo Receivers for Interleave-Division Multiple-Access Systems

Bogdan Cristea, Daniel Roviras, and Benoît Escrig

***Abstract*—In this paper several turbo receivers for Interleave-Division Multiple-Access (IDMA) systems will be discussed. The multiple access system model is presented first. The optimal, Maximum A Posteriori (MAP) algorithm, is then presented. It will be shown that the use of a precoding technique at the emitter side is applicable to IDMA systems. Several low complexity Multi-User Detector (MUD), based on the Gaussian approximation, will be next discussed. It will be shown that the MUD with Probabilistic Data Association (PDA) algorithm provides faster convergence of the turbo receiver. The discussed turbo receivers will be evaluated by means of Bit Error Rate (BER) simulations and EXtrinsic Information Transfer (EXIT) charts.**

***Index Terms*—Multiple access systems, random permutations, turbo receivers, multipath channels, multiple-access interference.**

I. INTRODUCTION

SEVERAL second- and third-generation cellular mobile networks are currently implementing Direct-Sequence Code-Division Multiple-Access (DS-CDMA) [1]. These DS-CDMA systems are known to suffer from Multiple-Access Interference (MAI) and Inter-Chip Interference (ICI). MUD techniques have been designed in order to cope with the effects of MAI and ICI [2]. These techniques jointly process the signals from all users to improve performance. Since the transceivers are also constituted with error correcting encoders and interleavers, turbo receivers can be implemented at the receiver side to further improve performance [3]. These receivers are usually constituted by a Soft-Input Soft-Output (SISO) MUD module and a SISO Decoder (DEC) module with judicious interleaving in between.

Beside the implementation of turbo receivers, the presence of interleavers in the transceivers has motivated the design of multiple access techniques based on interleavers [4]. Access methods such as IDMA have been proposed as an alternate approach to DS-CDMA [5]–[7]. In IDMA systems, interleavers are employed as the only means of user separation. Since IDMA is a special form of DS-CDMA, it inherits many advantages of DS-CDMA (dynamic channel sharing, mitigation of cross-cell interference, asynchronous transmission, ease of cell planning, and robustness against fading). It also allows the use of low-complexity MUD techniques applicable to systems with large numbers of users in multipath channels. During the last few years, many researches have been undertaken around the topic of IDMA systems. These studies focused on power allocation [8]–[10] and the design of new IDMA systems [11]–[13]. Apart from these topics, several techniques have been proposed to optimize the design of turbo receivers for IDMA systems. Since the early works of [6], [14], the turbo receivers relied on an optimization technique called Gaussian Chip Detector (GCD) to lower the complexity of the reception algorithm [15]. This technique is also known as Elementary Signal Estimation (ESE). The GCD principle consists in approximating the ICI and the MAI by Gaussian random variables. This assumption greatly simplifies the computation of extrinsic information in SISO MUD. This Gaussian Approximation (GA) has been introduced first in [16]. The equivalence between the GA [16] and the GCD [15] has been shown in [17]. Further improvements in the IDMA receivers design have been achieved recently. In [18], several low-cost MUD techniques have been compared. It has been shown that chip extrinsic information, a posteriori information and bit extrinsic information were providing different tradeoffs between performance and complexity. In [19], a new decoding strategy has been obtained by replacing the classical convolutional encoder at the emitter side by a convolutional code followed by a repetition code. In [20], a simplified ESE algorithm has been proposed together with a new Orthogonal Frequency Division Multiplex (OFDM)-IDMA system, and the joint code-rate and spreading-factor optimization has been addressed in [21]. In [22], optimizations at the system level have been made using two semi-analytical tools, namely, the large-system performance approximation and the EXIT chart technique [23].

Regarding turbo receiver design, almost concurrently to [15], another low-complexity turbo receiver algorithm has been proposed in [24]. The receiver architecture is based on the PDA algorithm. The PDA has been originally proposed in [25] and later used for equalization in [26]. Further refinements of the PDA algorithm have been proposed in the frame of Multiple-Input Multiple-Output (MIMO) systems [27], [28]. The basic idea of PDA is to approximate the mixture of Gaussian distribution at the receiver input with a single Gaussian distribution.

In this paper, a new turbo receiver, based on the PDA algorithm, will be proposed for IDMA systems. We will also emphasize some interesting properties of the IDMA systems

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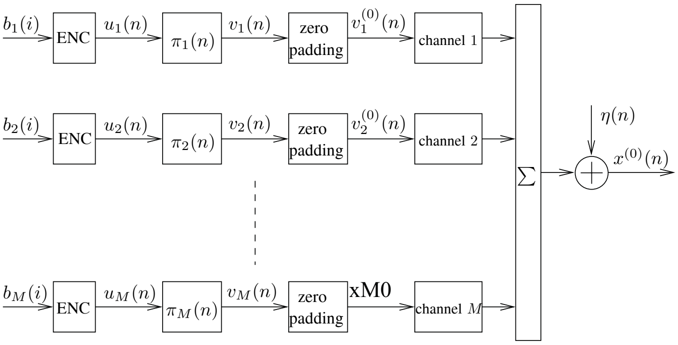

Fig. 1. Model of IDMA system.

in order to show how its performance can be improved. The remaining of the paper is organized as follows. The system model will be presented in section II. Section III discusses several choices for turbo multi-user receivers. Simulation results will be presented in section IV. Finally, conclusions are drawn in section V.

II. SYSTEM MODEL

In this part, we present the IDMA system and give the expression of the IDMA signal at the receiver input. The time-discrete model of the IDMA system is given in Fig. 1. We consider the IDMA system in a quasi-synchronous configuration, which means that the relative delays between users are small enough to be absorbed in a common channel order L.

The input signal of the μ-th user is a Binary Phase Shift Keying (BPSK) modulated signal, $b_\mu(i) \in \{-1, +1\}$, i being the time index. Then, the input signal enters a repetition encoder (ENC) with a fixed-pattern scrambler $\varphi(n)$ with N_s values in the set $\{-1, +1\}$. For example, the scrambler pattern, $\varphi(n)$, can be a sequence of equally balanced symbols of $+1$ and -1: $\begin{bmatrix}+1 & -1 & +1 & -1 & \cdots\end{bmatrix}$.

The output signal of the encoder is interleaved by a block interleaver, defined by a random permutation [4] , $\pi_\mu(n)$, different for each user. Users' orthogonality and spreading effect are obtained using only random interleavers. So, each user has a different random interleaver of period N (the same period for all users), chosen much greater than the length of the scrambler pattern N_s: $N \gg N_s$. The design of orthogonal interleavers was discussed in [29] and the spreading effect of random permutations in [4].

If the pattern of the repetition encoder has values in the set $\{-1, +1\}$, then the IDMA system resembles with a DS-CDMA system [30], where the role of channelization and scrambling codes is played by random interleavers. Thus, in the following, the samples of the interleaved signal will be called "chips".

The interleaved signal is passed through a Zero Padding (ZP) device whose role is to add zeros periodically into the interleaved signal. Thus, each frame of N nonzero chips ends with L zeros. The goal of this approach is to cancel the inter-block interference, where a block is represented by N successive chips at the random interleaver output [31]. After the ZP, the signal of the μ-th user is:

$$\mathbf{v}_\mu^{(0)}(k) = \mathbf{T}_{\text{zp}} \mathbf{v}_\mu(k) \tag{1}$$

where

$$\mathbf{T}_{\text{zp}} = \begin{bmatrix} \mathbf{I}_N \\ \mathbf{0}_{L\times N} \end{bmatrix} \tag{2}$$

is the ZP matrix, with $\mathbf{I}_N$ the unit matrix and $\mathbf{0}_{L\times N}$ the all zero matrix with L lines and N columns, and

$$\mathbf{v}_\mu(k) = \begin{bmatrix} v_\mu(kN+1) & v_\mu(kN+2) & \cdots & v_\mu(kN+N) \end{bmatrix}^{\mathcal{T}} \tag{3}$$

is a column vector represented by N interleaved chips. The notation ${}^{\mathcal{T}}$ stands for the transpose of some vector and the superscript ${}^{(0)}$ is used to denote ZP signals.

The propagation channels are assumed slow fading, constant for the whole transmission duration. The channel impulse response of the μ-th user is:

$$h_\mu(n) = \sum_{l=0}^{L} h_l^{(\mu)} \delta(n-l) \tag{4}$$

where $h_l^{(\mu)}$ are the channel coefficients and $\delta(n)$ is the Kronecker function. Without lost of generality and in order to simplify the presentation of the IDMA system, we assume that the channels have real coefficients. The case when the channel is complex can be reduced to the real channel case using complex to real matrix transformations [32, p. 30].

Thus, the received signal can be written as [31]:

$$\mathbf{x}^{(0)}(k) = \sum_{m=1}^{M} \Big(\mathbf{H}_0^{(m)} \mathbf{v}_m^{(0)}(k) + \mathbf{H}_1^{(m)} \mathbf{v}_m^{(0)}(k-1) \Big) + \boldsymbol{\eta}(k) \tag{5}$$

where M is the number of users and $\boldsymbol{\eta}(k)$ is a column vector of length $(N+L)$ representing the Additive White Gaussian Noise (AWGN). The channel matrices, $\mathbf{H}_0^{(m)}$ and $\mathbf{H}_1^{(m)}$, are defined as:

$$\mathbf{H}_0^{(m)} = \begin{bmatrix} h_0^{(m)} & 0 & \cdots & 0 & 0 \\ h_1^{(m)} & h_0^{(m)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & h_L^{(m)} & \cdots & h_1^{(m)} & h_0^{(m)} \end{bmatrix} \tag{6}$$

$$\mathbf{H}_1^{(m)} = \begin{bmatrix} 0 & \cdots & h_L^{(m)} & \cdots & h_2^{(m)} & h_1^{(m)} \\ 0 & \cdots & 0 & \cdots & h_3^{(m)} & h_2^{(m)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{7}$$

Knowing that $\mathbf{H}_1^{(m)} \mathbf{T}_{\text{zp}} = \mathbf{0}_{(N+L)\times N}$ $m \in \{1, 2, \ldots, M\}$, (5) can be rewritten as:

$$\mathbf{x}^{(0)}(k) = \sum_{m=1}^{M} \mathbf{H}^{(m)} \mathbf{v}_m(k) + \boldsymbol{\eta}(k) \tag{8}$$

where $\mathbf{H}^{(m)} = \mathbf{H}_0^{(m)} \mathbf{T}_{\text{zp}}$ is an $(N+L) \times N$ band matrix:

$$\mathbf{H}^{(m)} = \begin{bmatrix} h_0^{(m)} & 0 & \cdots & 0 \\ h_1^{(m)} & h_0^{(m)} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & h_0^{(m)} \\ 0 & 0 & \cdots & h_1^{(m)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & h_L^{(m)} \end{bmatrix} \tag{9}$$

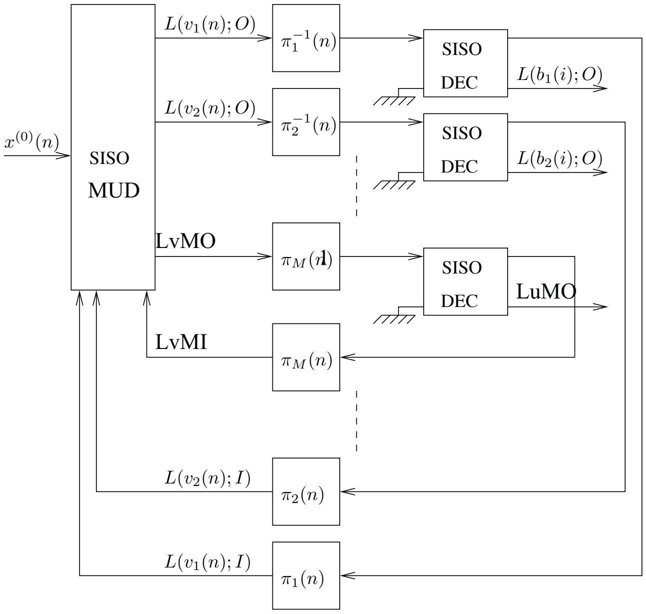

Fig. 2. Turbo multi-user receiver for IDMA systems.

Note that, in (8), the inter-block interference, between $\mathbf{v}_m^{(0)}(k)$ and $\mathbf{v}_m^{(0)}(k-1)$, is eliminated thanks to the inserted zeros. Thus, each emitted block of N chips, $\mathbf{v}_\mu(k)$, can be recovered using the received block of samples $\mathbf{x}^{(0)}(k)$, without taking into account the interference between adjacent blocks of chips. This approach is useful mainly when turbo receivers are used. This will be the topic of the next section.

III. TURBO MULTI-USER RECEIVERS

This section will present several turbo multi-user receivers suitable to use in IDMA systems. The general structure of a turbo multi-user receiver is presented in Fig. 2. The turbo multi-user receiver relies on the iterative exchange of extrinsic information between a SISO MUD module and the SISO DEC modules of all users (Fig. 2). The SISO MUD module is used to cope with the effects of propagation channels and with the MAI. The outputs of the SISO MUD, $L(v_m(n);O)$, are used, after deinterleaving, as inputs for the SISO DEC. The SISO DEC module was described in [33] so, only different choices for the SISO MUD will be discussed in the following.

A. Maximum A Posteriori algorithm

The MAP algorithm used in SISO MUD is presented here without the use of the ZP technique (see section II). Thus, we will drop in our notations the superscript $^{(0)}$. In order to obtain the initial conditions for the forward and backward recusions of the MAP algorithm, one can use the hypothesis that the multipath channels of all users are represented by a trellis with known initial state (zero state) and with an unknown final state. So, when using the MAP algorithm, the ZP technique presented in section II is no longer necessary and only a given number of known symbols must be transmitted at the beginning of each block in order to ensure a known initial state of the channel.

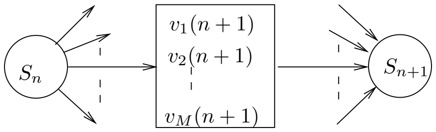

Fig. 3. Transition between states of the hyper-trellis representing the channels of all users.

In the SISO MUD, the MAP algorithm is applied on a hyper-trellis represented by the L preceding entries in the channels of all users [33]:

$$\begin{aligned} S_n \;=\; & (v_1(n),\ v_1(n-1),\ ...,\ v_1(n-L+1),\\ & v_2(n),\ v_2(n-1),\ ...,\ v_2(n-L+1),\\ & \dots\dots\dots\dots\dots\dots\dots\dots\\ & v_M(n),\ v_M(n-1),\ ...,\ v_M(n-L+1)) \end{aligned}$$

where M is the number of users and L is the order of multipath channels.

Starting from a given state, S_n, there are 2^M output branches corresponding to all possible inputs from all M users (Fig. 3):

$$(v_1(n+1),\ v_2(n+1),...,\ v_M(n+1)) \tag{10}$$

The Logarithm of Likelihood Ratio (LLR) of the chips sent in the multipath channel of the μ-th user, $\Lambda(v_\mu(n))$, can be computed using the relation [33]:

$$\Lambda(v_\mu(n)) = \ln\frac{P\left(v_\mu(n)=+1/\mathbf{x}\right)}{P\left(v_\mu(n)=-1/\mathbf{x}\right)} \tag{11}$$

$$= \ln\frac{\sum_{(S_{n-1},S_n)}\alpha_{n-1}(S_{n-1})\gamma_\mu^{(1)}(x(n),S_{n-1},S_n)\beta_n(S_n)}{\sum_{(S_{n-1},S_n)}\alpha_{n-1}(S_{n-1})\gamma_\mu^{(-1)}(x(n),S_{n-1},S_n)\beta_n(S_n)} \tag{12}$$

where $\mu\in\{1,2,...,M\}$ and $n\in\{1,2,...,N\}$.

The forward recursion can be written as:

$$\alpha_n(S_n) = \sum_{(S_{n-1})}\sum_{i\in\{-1,1\}}\alpha_{n-1}(S_{n-1})\gamma_\mu^{(i)}(x(n),S_{n-1},S_n) \tag{13}$$

$\forall n\in\{1,2,...,N\}$ with the initial condition:

$$\alpha_0(S_0) = \begin{cases}1 & S_0=0,\\ 0 & \forall S_0\neq 0\end{cases} \tag{14}$$

and the backward recusion as:

$$\beta_n(S_n) = \sum_{(S_{n+1})}\sum_{i\in\{-1,1\}}\beta_{n+1}(S_{n+1})\gamma_\mu^{(i)}(x(n+1),S_n,S_{n+1}) \tag{15}$$

$\forall n\in\{0,1,...,N-1\}$ with the initial condition:

$$\beta_N(S_N) = 1\ \forall S_N \tag{16}$$

The probability of transition from state S_{n-1} to state S_n is:

$$\gamma_\mu^{(i)}(x(n),S_{n-1},S_n) = \frac{p(x(n)/S_n,S_{n-1})P(v_\mu(n)=i/S_n,S_{n-1})P(S_n/S_{n-1})}{P(x(n)/x(1),x(2),\dots,x(n-1))} \tag{17}$$

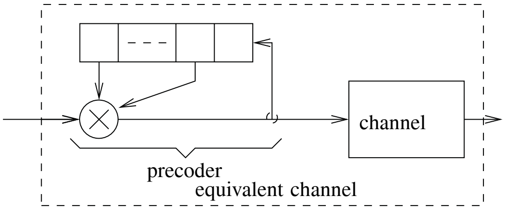

Fig. 4. Equivalent recursive channel.

where

$$P(v_\mu(n) = i/S_n, S_{n-1}) = \begin{cases} 1 & \text{if } S_{n-1} \xrightarrow{v_\mu(n)=i} S_n \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

and $P(S_n/S_{n-1})$ is the *a priori* probability of chips emitted by all users at a given time instant n. With the hypothesis that the chips emitted by all users are independent, one can obtain:

$$P(S_n/S_{n-1}) = \Pi_{m=1}^{M} P(v_m(n)) \quad (19)$$

The *a priori* probability, $P(v_m(n))$, can be computed using the *a priori* information, $L(v_m(n); I)$:

$$P(v_m(n)) = \frac{\exp\left(0.5 \cdot (v_m(n) + 1) \cdot L(v_m(n); I)\right)}{1 + \exp\left(L(v_m(n); I)\right)} \quad (20)$$

where the *a priori* information is defined as:

$$L(v_m(n); I) = \ln \frac{P\left(v_m(n) = +1\right)}{P\left(v_m(n) = -1\right)} \quad (21)$$

Thus, one can compute the LLR of emitted chips using (12) and then the extrinsic information:

$$L(v_\mu(n); O) = \Lambda(v_\mu(n)) - L(v_m(n); I) \quad (22)$$

It is worth to emphasize that IDMA systems with turbo MUD are a generalization of coded transmissions with turbo equalization [34]. Since the turbo equalizer performances can be improved by implementing a precoder (recursive convolutional code of coding rate 1) at the emitter side [35], [36], one can use the same approach to improve performance in IDMA systems. When a precoder is used at the channel input, an equivalent recursive channel is obtained (Fig. 4). In this case, in the SISO MUD, only the hyper-trellis is changed and the MAP algorithm will be described by the same relations as above. The choice of the precoder generator polynomial can affect IDMA system performance, but this issue will not be discussed here.

Note that the serial concatenation of a repetition code with a recursive code resembles with Repeat-Accumulate (RA) codes [37]. These codes have performance close to turbo codes [37] and thus their use in IDMA systems is suitable due to their very good performances at low Signal to Noise Ratio (SNR). Further improvements in performance can be achieved by investigating the design of RA codes and/or of interleavers. This issue goes beyond the scope of our paper and will be kept for further studies.

However, the MAP algorithm is too complex for most practical application and thus lower complexity MUD algorithms are needed. These lower complexity algorithms will be discussed in the next subsections.

B. Gaussian Chip Detector

In order to replace the MAP algorithm in the SISO MUD, lower complexity MUD algorithms have been proposed [15]. These algorithms compute directly the extrinsic information $L(v_\mu(n); O)$ using

$$L(v_\mu(n); O) = \ln \frac{p\left(\mathbf{x}^{(0)}/v_\mu(n) = +1\right)}{p\left(\mathbf{x}^{(0)}/v_\mu(n) = -1\right)} \quad (23)$$

and relying on the assumption that the ICI and the MAI are both Gaussian random variables. The above relation can be proven using Bayes's rule in (11).

Thus, in (8), one can separate the chips emitted by the μ-th user at instant n, $v_\mu(n)$, from the ICI and the MAI:

$$\mathbf{x}^{(0)} = \mathbf{h}^{(\mu)}(n) v_\mu(n) + \boldsymbol{\zeta}_\mu(n) \quad (24)$$

where $\mathbf{h}^{(\mu)}(n)$ is the n-th column of the matrix $\mathbf{H}^{(\mu)}$ (9) $\mu \in \{1, 2, \ldots, M\}, n \in \{1, 2, \ldots, N\}$ and

$$\boldsymbol{\zeta}_\mu(n) = \sum_{\substack{m=1 \\ (m \neq \mu)}}^{M} \sum_{\substack{i=1 \\ \&(i \neq n)}}^{N} \mathbf{h}^{(m)}(i) v_m(i) + \boldsymbol{\eta} \quad (25)$$

is the ICI and the MAI. For mathematical convenience we have chosen to discard in (8) the index k of the vectors $\mathbf{x}^{(0)}(k)$, $\mathbf{v}_m(k)$ and $\boldsymbol{\eta}(k)$.

Under the assumption that the ICI and the MAI are Gaussian random variables, $\boldsymbol{\zeta}_\mu(n)$ is a vector of Gaussian random variables. These random variables are correlated after multipath propagation. Thus, with (24), it can be shown that the extrinsic information $L(v_\mu(n); O)$ can be written as [15]:

$$L(v_\mu(n); O) = 2\mathbf{h}^{(\mu)}(n)^{\mathcal{T}} \operatorname{Cov}^{-1}\left[\boldsymbol{\zeta}_\mu(n)\right] \left(\mathbf{x}^{(0)} - \operatorname{E}\left[\boldsymbol{\zeta}_\mu(n)\right]\right) \quad (26)$$

where $\operatorname{Cov}\left[\boldsymbol{\zeta}_\mu(n)\right]$ is the covariance matrix of the random vector $\boldsymbol{\zeta}_\mu(n)$:

$$\operatorname{Cov}\left[\boldsymbol{\zeta}_\mu(n)\right] = \sum_{\substack{m=1 \\ (m \neq \mu)}}^{M} \sum_{\substack{i=1 \\ \&(i \neq n)}}^{N} \mathbf{h}^{(m)}(i) \mathbf{h}^{(m)\mathcal{T}}(i) \operatorname{Var}\left[v_m(i)\right] + \sigma^2 \mathbf{I}_{N+L} \quad (27)$$

and $\operatorname{E}\left[\boldsymbol{\zeta}_\mu(n)\right]$ is the mathematical expectation of the random vector $\boldsymbol{\zeta}_\mu(n)$:

$$\operatorname{E}\left[\boldsymbol{\zeta}_\mu(n)\right] = \sum_{\substack{m=1 \\ (m \neq \mu)}}^{M} \sum_{\substack{i=1 \\ \&(i \neq n)}}^{N} \mathbf{h}^{(m)}(i) \operatorname{E}\left[v_m(i)\right] \quad (28)$$

The mathematical expectation of the emitted chip, $\operatorname{E}\left[v_m(i)\right]$, can be computed from the *a priori* information of the emitted chip $L\left(v_m(i); I\right)$:

$$\operatorname{E}\left[v_m(i)\right] = P\left(v_m(i) = +1\right) - P\left(v_m(i) = -1\right) \quad (29)$$

$$= \tanh\left(\frac{L\left(v_m(i); I\right)}{2}\right) \quad (30)$$

where $L\left(v_m(i); I\right)$ is defined in (21).

From (30), the variance of the emitted chip, $\operatorname{Var}\left[v_m(i)\right]$, is obtained using:

$$\operatorname{Var}\left[v_m(i)\right] = 1 - \operatorname{E}^2\left[v_m(i)\right] \quad (31)$$

So, with (26), the extrinsic information of each emitted chip, $L(v_\mu(n);O)$, is computed using the knowledge of the received vector, $\mathbf{x}^{(0)}$, the channel attenuations, the *a priori* information, $L(v_\mu(n);I)$, and the variance of the AWGN, σ^2. The SISO MUD based on (26) is a Gaussian Chip Detector. This algorithm has been presented previously in [15].

C. Simplified Gaussian Chip Detector

The interest of this subsection is to unveil the link between the different MUD algorithms presented in [15]. In [15], the Log-Likelihood Ratio Combining (LLRC) is in fact equivalent to the simplified GCD whereas the Joint Gaussian Combining (JGC) is equivalent to the standard GCD.

Thus, the above described GCD can be further simplified by considering that the covariance matrix, $\operatorname{Cov}\left[\boldsymbol{\zeta}_\mu(n)\right]$, is a diagonal matrix. This approach is justified since, in general, the terms on the diagonal of the covariance matrix have values much greater than the other terms. The same approach is used in DS-CDMA systems, where the covariance matrix of spreading codes is assumed a diagonal matrix [2, p. 248].

Using a diagonal covariance matrix in (26), the expression of the extrinsic information at the SISO MUD output becomes:

$$L(v_\mu(n);O) = 2\sum_{l=0}^{L} h_l^{(\mu)} \frac{y^{(0)}(n+l) - \left(\mathrm{E}\left[\mathbf{x}^{(0)}\right]\right)_{n+l} + h_l^{(\mu)}\,\mathrm{E}\left[v_\mu(n)\right]}{\left(\operatorname{Cov}\left[\mathbf{x}^{(0)}\right]\right)_{n+l} - h_l^{(\mu)2}\operatorname{Var}[v_\mu(n)]} \tag{32}$$

where $\left(\mathrm{E}\left[\mathbf{x}^{(0)}\right]\right)_{n+l}$ is the $(n+l)$-th element of the vector $\mathrm{E}\left[\mathbf{x}^{(0)}\right]$ defined as:

$$\mathrm{E}\left[\mathbf{x}^{(0)}\right] = \sum_{m=1}^{M}\sum_{i=1}^{N} \mathbf{h}^{(m)}(i)\,\mathrm{E}\left[v_m(i)\right] \tag{33}$$

and $\left(\operatorname{Cov}\left[\mathbf{x}^{(0)}\right]\right)_{n+l}$ is the $(n+l)$-th element on the diagonal of the matrix $\operatorname{Cov}\left[\mathbf{x}^{(0)}\right]$, defined as:

$$\operatorname{Cov}\left[\mathbf{x}^{(0)}\right] = \sum_{m=1}^{M}\sum_{i=1}^{N} \mathbf{h}^{(m)}(i)\mathbf{h}^{(m)T}(i)\operatorname{Var}[v_m(i)] + \sigma^2\mathbf{I}_{N+L} \tag{34}$$

In order to prove (32) we have used the fact that in (26) the vector $\mathbf{h}^{(\mu)}(n)$ has only $L+1$ nonzero entries.

With this approach great complexity reduction is achieved since, instead of having to compute the inverse of the covariance matrix as in (26), only simple divisions are used in (32). Thus, a second SISO MUD, called simplified GCD, is obtained.

The following section generalizes the previously presented methods, based on the GA approach.

D. Probabilistic Data Association algorithm

In this subsection, we shall present a turbo receiver based on the PDA algorithm and the link between this algorithm and the GCD algorithms - the standard GCD and the simplified GCD.

The PDA algorithm relies also on the assumption that the influence of the ICI and the MAI can be approximated by a single Gaussian random variable. This approximation is called probabilistic data association [24]. The SISO MUD and the SISO DEC are exchanging extrinsic informations, $L(v_\mu(n);O)$, using either (26) or (32). The mathematical expectation of an emitted chip, $\mathrm{E}\left[v_m(i)\right]$, is determined by (29) [24]:

$$\begin{aligned}\mathrm{E}\left[v_m(i)\right] &\approx P\left(v_m(i) = +1/\mathbf{x}^{(0)}\right) - P\left(v_m(i) = -1/\mathbf{x}^{(0)}\right)\\ &\approx \tanh\left(\frac{\Lambda\left(v_m(i)\right)}{2}\right)\end{aligned} \tag{35}$$

where

$$\Lambda\left(v_m(i)\right) = L\left(v_m(i);I\right) + L\left(v_m(i);O\right) \tag{36}$$

is the LLR of emitted chip $v_m(i)$ (11).

Moreover, in the PDA version of a SISO MUD, several local iterations are realized before the extrinsic information is delivered to the SISO DEC.

Thus, the PDA algorithm used in the SISO MUD can be synthesized as follows:

1) First iteration: the extrinsic information at the MUD output is zero
$$L\left(v_m(i);O\right) = 0$$
$m \in \{1,2,\ldots,M\}, i \in \{1,2,\ldots,N\}$;
2) Based on the *a priori* information at the MUD input, $L\left(v_m(i);I\right)$, and the extrinsic information at the MUD output, $L\left(v_m(i);O\right)$, the mathematical expectation of the emitted chips is obtained as (36), (35):
$$\mathrm{E}\left[v_m(i)\right] \approx \tanh\left(\frac{L\left(v_m(i);I\right) + L\left(v_m(i);O\right)}{2}\right)$$
3) With the vector of received samples, $\mathbf{x}^{(0)}$, the mathematical expectation $\mathrm{E}\left[v_m(i)\right]$ and the variance $\operatorname{Var}\left[v_m(i)\right] = 1 - \mathrm{E}^2\left[v_m(i)\right]$, the extrinsic information, $L(v_m(i);O)$, can be obtained with (26) or (32);
4) Last iteration: stop algorithm, otherwise: go to step 2.

So, the SISO MUD using PDA algorithm without any local iteration is equivalent with a SISO MUD based on the GA (a GCD or a simplified GCD). The main difference between the PDA algorithm and the GCD algorithms is that additional local iterations at the SISO MUD with PDA are used to better cancel the ICI and the MAI. The estimation of the mean of the Gaussian random variable (see (28) and (30)) is realized several times, using both extrinsic (23) and *a priori* information (21).

Note that expression (35) is very close to expression (30). However, the SISO MUD input is not constituted by the *a priori* information $L(v_m(i);I)$, but rather by the LLR $\Lambda(v_m(i))$. This approach is justified because the PDA algorithm does not accept yet the *a priori* information at its input. The consequence is that the turbo receiver based on PDA is a sub-optimal receiver since there is a possibility that the algorithm finds a non-optimal solution. Such problems can happen especially at early iterations because the remaining ICI and MAI have still large values. To overcome the problem, less PDA SISO MUD iterations should be allowed at early turbo iterations. A more elegant solution will be to adjust adaptively the number of iterations in the PDA SISO MUD [24]. We

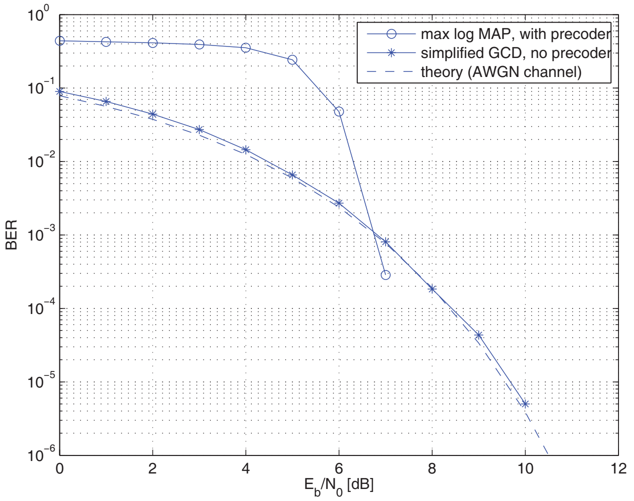

Fig. 5. Performances of the IDMA system with $M = 2$ users when a precoder is used.

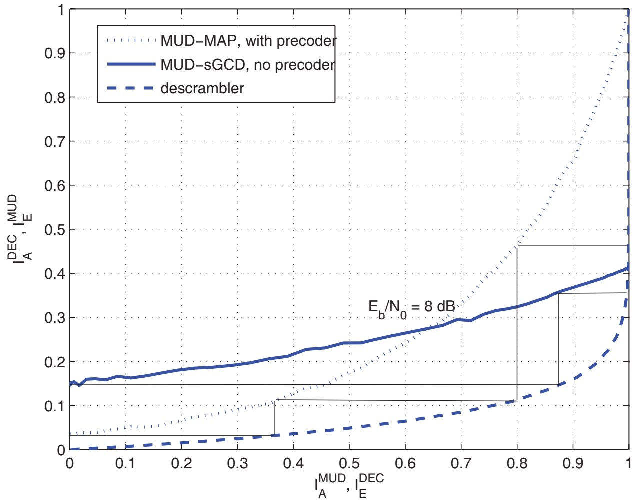

Fig. 6. EXIT chart of the turbo multi-user receiver for $M = 2$ users and at a SNR $\frac{E_b}{N_0} = 8\ dB$.

propose to design this adaptation step with an EXIT charts analysis [23]. This analysis and the performance of IDMA turbo receivers will be presented in the next section.

IV. SIMULATION RESULTS

In this section the performance of IDMA receivers is evaluated by means of BER simulations and EXIT charts.

The system model is depicted in Fig. 1. The repetition encoder is a fixed-pattern scrambler $\varphi(n)$ constituted with a sequence of equally balanced symbols of $+1$ and -1, with $N_s = 16$. Random interleavers are different for each user and have the same period $N = 38400$ (the length of scrambling codes in 3GPP [30]). Propagation channels are random, with Rayleigh distributed coefficients and order $L = 3$.

The first set of simulations aims at showing that precoders can improve the performance of IDMA systems by evaluating the performance of turbo multi-user receivers, without and with precoder. The precoder has the generator polynomial $p(D) = 1 + D$ and the SISO MUD module uses the max log MAP algorithm. We chose a configuration with $M = 2$ users in our system. In this case, the number of states of the hyper-trellis used in the MAP algorithm is $2^{2 \cdot 3} = 64$. For $M \geq 3$ the number of states is greater or equal than $2^{3 \cdot 3} = 256$ and thus the complexity of the MAP algorithm becomes too high for this type of multipath channels (of order $L = 3$). When no precoder is used, the simplified GCD is used by the SISO MUD module and the results are compared with the theoretical performance in an AWGN channel (Fig. 5).

Thus, it can be seen that for a SNR large enough (in our case $\frac{E_b}{N_0} \geq 7\ dB$) the use of the precoder can ensure better performance than in the case when no precoder is used. The simplified GCD has performace close to the theoretical performance, that is the MAI and the multipath interferrence are completely eliminated even if the SISO MUD is based on the lowest complexity algorithm (the simplified GCD).

The EXIT diagrams of the turbo multi-user receiver are presented in Fig. 6. When no precoder is used, the receiver performance is limited by the SISO MUD module, since its mutual extrinsic information has its maximum value at $I_E^{MUD} = 0.4$. Here, for simplicity reasons, we have chosen to use the simplified GCD, but, when the max log MAP algorithm is used, the same transfer characteristic of the SISO MUD module is expected, since the simplified GCD already gives the best possible performance (Fig. 5). When the precoder is used, the mutual extrinsic information can reach the maximum possible value, $I_E^{\mathrm{MUD}} = 1$, and thus the performance of the turbo multi-user receiver are no longer limited by the SISO MUD module.

So, the use of the precoder at each channel input gives the best possible performance when the SNR is large enough (Fig. 5). However, it should be noted that with a precoder at the emission side, the algorithms used in the SISO MUD module need to be adapted in order to be able to decode both the precoder and the channel. Such an adaptation is possible when the MAP algorithm is used, but for the other lower complexity algorithms (based on the GA) this adaptation has not been yet realized to our knowledge.

Secondly, the performance of turbo multi-user receivers has been studied when the PDA algorithm is used. The IDMA system has been tested with $M = 8$ users. The maximum number of users allowed in the system depends on the length of the scrambler pattern, $N_s = 16$, but also on the multipath channel characteristics (number of multipaths, delay profile, etc.). For comparison purposes, two SISO MUDs are considered: the simplified GCD and the MUD with PDA algorithm. The PDA algorithm is based on the simplified GCD algorithm with a second loop between the output and the input of the SISO MUD.

The EXIT charts were used to study the turbo multi-user receiver when the PDA algorithm is employed (Fig. 7). When the simplified GCD is used, good performance can be achieved by increasing the number of iterations. On the other hand, when the detector based on PDA algorithm is used only, there is very early in the iterative process an intersection point between the two transfer caracteristics (of the SISO MUD with PDA algorithm and of the SISO DEC). So, poor performance

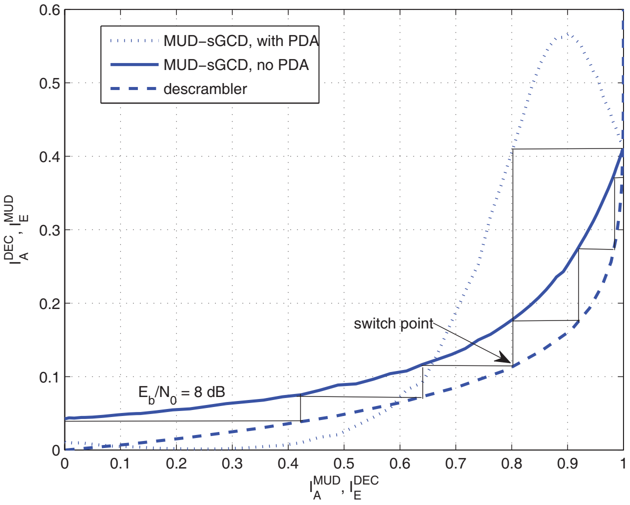

Fig. 7. EXIT chart of the turbo multi-user receiver for $M = 8$ users and at a SNR $\frac{E_b}{N_0} = 8\ dB$.

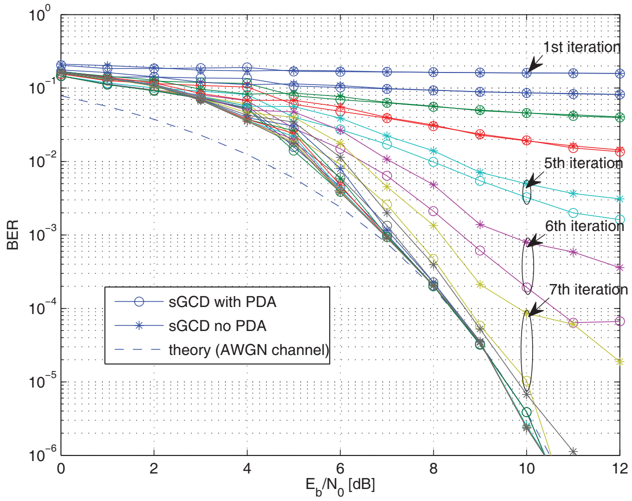

Fig. 8. Performances of the IDMA system with simplified GCD and PDA algorithm for $M = 8$ users.

is achieved in this case even if the number of iterations is increased. A possible explaination of the latter situation is that, when the mutual *a priori* information, I_A^{MUD}, is very low, the estimation of the emitted chips (contained in the *a priori* information) is so poor that adding the extrinsic information to the existing *a priori* information has an adverse effect and worsens the performance.

However, when the mutual *a priori* information at the SISO MUD input is large enough, $I_A^{MUD} > 0.8$, it can be seen that the SISO MUD with PDA algorithm provides a larger mutual extrinsic information, I_E^{MUD} than in the case of the simplified GCD (Fig. 7). A greater mutual extrinsic information means a better estimation of the emitted chips and thus better performance. Thus, in our approach, we have chosen to use for the first 3 iterations the simplified GCD and then to switch to the SISO MUD with PDA algorithm in order to increase the quality of the mutual extrinsic information at the SISO MUD output.

Note that, when using the detector with PDA algorithm, the mutual extrinsic information, I_E^{MUD}, has a peak at approximatively $I_A^{MUD} = 0.9$ and then decreases to the same value as for the simplified GCD. This peak seems to be characteristic for the PDA algorithm and is due to the fact that the PDA algorithm uses a posteriori information (extrinsic information plus *a priori* information) instead of *a priori* information as in classical scheme.

At the end of the iterative process the mutual extrinsic information provided by the simplified GCD and the MUD with PDA algorithm is the same $I_E^{MUD} = 0.4$. Here there is an explanation since, in both cases, the cancelling of the ICI and MAI is based on the *a priori* information at the MUD input. Thus, when the mutual *a priori* information reaches its maximum value $I_A^{MUD} = 1$, further improvements of the mutual extrinsic information at the MUD output are no longer possible.

We have also used BER simulations to study the two SISO MUDs (Fig. 8). Almost the same performances are obtained when the simplified GCD and the detector with PDA algorithm are employed. The main advantage of the proposed SISO MUD with PDA algorithm is that it accelerates the convergence of the iterative reception algorithm, since the best possible performance (in AWGN channel) is achieved using a smaller number of iterations.

So, with the proposed SISO MUD with PDA algorithm, faster convergence of the turbo receiver is achieved, while the only modification in the previously known SISO MUDs is to use several local iterations for each global iteration in the turbo multi-user receiver.

V. CONCLUSION

In this paper several turbo receivers were discussed in order to improve IDMA system performance. First, the optimal, MAP algorithm was presented. It was shown that the system performance can be improved using a precoder before the permuted chips are emitted into the multipath channels. Further, we have underlined that the mono-user transmission chain with precoder ressembles with RA codes structure, which could explain the good performance achieved when a precoder is used. Two low complexity SISO MUDs, based on the GA, were also discussed for the case when no precoder is used. We have shown that previously proposed MUDs can be derived one from the other. A new turbo receiver, based on the PDA algorithm, was proposed for IDMA systems. The interest of the PDA approach is twofold. First, the PDA approach provides an unified view of existing low complexity MUDs for IDMA systems. Second, the PDA approach can be used to design turbo receivers with faster convergence.

Further work needs to be done in order to adapt lower complexity MUD algorithms for the case when a precoder is used at the emitter side. Another interesting direction of research should be the use of RA codes in IDMA systems.

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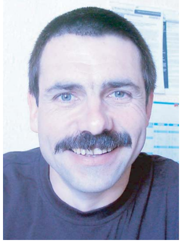

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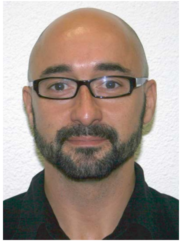

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